Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 15: Calculus I

15.1 Learning Intentions

After this week's lesson you will be able to;

- Explain what is meant by differentiation
- Differentiate a function from first principles.
- Differentiate polynomials by rule.
- Differentiate products and quotients.
- Differentiate composite functions.

15.2 Specification

5.2 Calculus	– find first and second derivatives of linear, quadratic and cubic functions by rule – associate derivatives with slopes and tangent lines – apply differentiation to • rates of change • maxima and minima • curve sketching	– differentiate linear and quadratic functions from first principles – differentiate the following functions • polynomial • exponential • trigonometric • rational powers • inverse functions • logarithms – find the derivatives of sums, differences, products, quotients and compositions of functions of the above form – apply the differentiation of above functions to solve problems

15.3 Chief Examiner's Report

B	8	26·6	53	8	Functions/rates of change
B	9	30·7	61	6	Functions/trigonometry/calculus

15.4 What is Differentiation?

To understand differentiation, we need to look at rates of change. If we have a linear constant rate of change then we can use this information to understand more about a particular function. From junior cycle we know a way of doing this (we also looked at this back in week 10) it is slope. We saw in the video that we cannot easily get the slope of a function that is non-linear, this is where differentiation comes in.

15.5 First Principles

What this refers to is the initial discovery regarding how to find the slope of a function at a given point.

We still retain the techniques from coordinate geometry; however, we have to do on very important thing... **ZOOM IN! (see video)**

<u>The 5 steps involved in differentiation by first principles:</u>

i) Find f(x):

ii) Find f(x+h):

iii) Get f(x+h)-f(x) and simplify:

iv) Divide by h (the gap between the two points):

Reduce h to almost 0 (find $\lim_{h \to 0} f(x)$):

15.6 Differentiation by rule

As first principles in quite time consuming, there are faster methods that have been developed. The first of this is the differentiating of a polynomial that is a sum/difference. The rule is:

$$\text{If } y = x^n, \text{ then } {}^{dy}/_{dx} = nx^{n-1}, \text{ where } n \in \mathbb{N}$$

Question:

If $y = x^2$

$dy/_{dx} = 2x^{2-1}$

$dy/_{dx} = 2x$

If $y = 4x^3$

Question:

If $y = 4x^3$

$dy/_{dx} = (3)4x^{3-1}$

$dy/_{dx} = 12x^2$

Question:

If $y = 3x$

$dy/_{dx} = 3x^{1-1}$

$dy/_{dx} = 3$

15.7 Product Rule

The above rule works if our function is a polynomial that is a sum of terms. However, if our function is a product, then we have a slightly different rule:

$\boldsymbol{y = (u)(v)}$

Then...

$\boldsymbol{dy/_{dx} = u\, dv/_{dx} + v\, du/_{dx}}$

Example:

If $y = (3x+2)(4x^2-5)$, find dy/dx

15.8 Quotient Rule

The function in question may also appear as a fraction (Quotient) in which case there are two options,

1) Use the Quotient rule:

If $y = \frac{u}{v}$

Then...

$$dy/dx = \frac{v\,du/dx - u\,dv/dx}{v^2}$$

2) Use the product rule with a small manipulation:

If $y = \frac{u}{v}$

Rearrange to

$y = (u)(v^{-1})$

and then use the product rule.

Example:

If $y = \frac{x^2-9}{x+3}$, find dy/dx

15.8 Quotient Rule

This is where we have a composite function to differentiate
In other words, this is where there is more than one thing happening within the function to differentiate:

$f(x) = [g(x)]^{index}$

Example:

$y = (x^2-3)^4$

15.10 Recap of the Learning Intentions

After this week's lesson you will be able to;

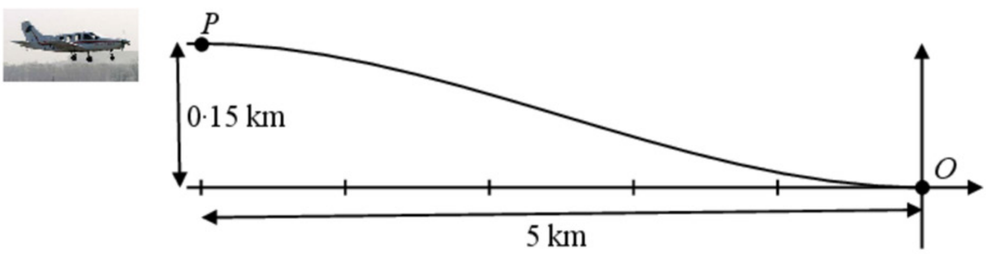
- Explain what is meant by differentiation
- Differentiate a function from first principles.
- Differentiate polynomials by rule.
- Differentiate products and quotients.
- Differentiate composite functions.

15.11 Homework Task

a. Differentiate the function $2x^2 - 3x - 6$ with respect to x from first principles.

Question 7 **50 marks**

A plane is flying horizontally at P at a height of 150 m above level ground when it begins its descent. P is 5 km, horizontally, from the point of touchdown 0. The plane lands horizontally at 0.



Taking O as the origin, $(x, f(x))$ approximately describes the path of the plane's descent wh
$f(x) = 0{\cdot}0024x^3 + 0{\cdot}018x^2 + cx + d$, $-5 \le x \le 0$, and both x and $f(x)$ are measured in km.

b. (ii) Find the value of f' (x), the derivative of f (x), when x=-4

15.8 Quotient Rule

a. A motoring magazine collected data on cars on a particular stretch of road. Certain details on 800 cars were recorded.

i. The ages of the 800 cars were recorded. 174 of them were new (less than 1 year old). Find the 95% confidence interval for the proportion of new cars on this road. Give your answer correct to 4 significant figures.

$$\frac{174}{800} - 1.96\sqrt{\frac{\frac{174}{800}\left(1-\frac{174}{800}\right)}{800}} \leq p \leq \frac{174}{800} + 1.96\sqrt{\frac{\frac{174}{800}\left(1-\frac{174}{800}\right)}{800}}$$

$$0.1889 \leq p \leq 0.2461$$

$$18.89\% \leq p \leq 24.61\%$$

ii. The data on the speeds of these 800 vehicles is normally distributed with an average speed of 87.3 km per hour and a standard deviation of 12 km per hour. What proportion of cars on this stretch of road would you expect to find travelling at over 95 km per hour?

$$z = \frac{x - \bar{x}}{\sigma}$$

$$z = \frac{95 - 87.3}{12} = 0.64167$$

$$p(Z \leq 0.64167) = 0.7389$$

$$p(z \geq 0.64) = 1 - 0.7389$$

$$= 0.2611 \text{ or } 26.11\%$$

When Conor rings Ciara's house, the probability that Ciara answers the phone is . 1/5

a. Conor rings Ciara's house once every day for 7 consecutive days. Find the probability that she will answer the phone on the 2nd, 4th, and 6th days but not on the other days.

$$\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}\times\frac{4}{5}=\frac{256}{78125}$$
$$=0.0032768$$

b. Find the probability that she will answer the phone for the 4th time on the 7th day.

$$\binom{6}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)=\frac{1280}{78125}$$
$$=0.016384$$

c. Conor rings her house once every day for n days. Write, in terms of n, the probability that Ciara will answer the phone at least once.

$$1-\left(\frac{4}{5}\right)^n$$